CONTROLLING CHAOS

Sudeshna Sinha

The Institute of Mathematical Sciences
Chennai

- Sinha, Ramswamy and Subba Rao: Physica D, vol. 43, p. 118
- Sinha, Physics Letts. A vol. 156, p. 475
- Ramswamy, Sinha, Gupte: Phys. Rev. E (Rapid Comm), vol. 57, p. 2507
- Sinha and Gupte: Phys. Rev. E (Rapid Comm), vol. 58, p. 5221
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A wide range of spatio-temporal dynamical phenomena occur in nature, in the laboratory and in numerical simulations :

- From Fixed points to Chaos
- From Coherence (such as in synchronised oscillator arrays)
  to Disorder (such as seen in fluid turbulence)

AIM :

Devise control strategies capable of achieving the desired type of spatio-temporal behaviour in complex systems

- Find techniques which direct strongly nonlinear, intrinsically chaotic systems on to regular targets
- Enhancement of spatio-temporal chaos also has important practical applications :

  Find algorithms to target Chaos

WISH LIST:

- To achieve control without having to monitor a large number of variables

  Must not be measurement intensive

- No extensive run-time computation

  Low control latency

- Robust with respect to noise

ADAPTIVE CONTROL

This is a class of efficient and easily implementable feedback methods targetting desired dynamical behaviour of wide-ranging complexity

Here a feedback loop drives the system parameter(s) to the value(s) required to achieve the desired state (target)

Implemented by augmenting the evolution equation for the dynamical system by an additional equation for the evolution of the parameter(s)

Consider a general N-dimensional nonlinear dynamical system described by the evolution equation

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}; \mu; t)$$

where $\mathbf{X} \equiv (X_1, X_2, \ldots X_N)$ are the state variables

μ is the parameter whose value determines the nature of the dynamics

The adaptive control is effected by the additional dynamics

$$\dot{\mu} = \gamma \ (\mathcal{P}^{\star} - \mathcal{P})$$

where

- $\mathcal{P}$ is a variable or property (which could be a function of several variables)
- $\mathcal{P}^{\star}$ is the target value of $\mathcal{P}$
- γ indicates the stiffness of control
- Error signal : $\mathcal{P}^{\star} - \mathcal{P}$
  Drives the system to the target state.
- Extension to many parameters is straightforward

The scheme is adaptive :

In the above procedure the parameters which determine the nature of the dynamics self-adjust or adapt themselves to yield the desired dynamics

Driven by the dynamic feedback

Prescription for adapative control :

- Property $\mathcal{P}$ should characterize the desired state well
  $\mathcal{P}$ should be distinctive : should be significantly different from states “nearby” in parameter space and phase space.
- The feedback can be spatial or temporal

  For instance:
  • If the desired target is a specific spatial pattern : the feedback should be spatial

  • If spatial periodicities are associated with concurrent temporal periodicity, e.g. spatio-temporal fixed points : either spatial or temporal feedback would be effective

- A parameter capable of effecting large dynamical changes is chosen to be controlled

  Feedback drives its value to a regime which naturally supports the desired spatio-temporal dynamics

- Property $\mathcal{P}$ must be simply defined without the explicit knowledge of the system's equations of motion i.e. without involving the explicit form of $\mathbf{F}(\mathbf{X})$:

  - Leads to considerable utility in experimental applications

  - Ensures low run-time computation

Controlling the Pinsky-Rinzel Model Neuron

Based on extensive physiological data, Pinsky and Rinzel developed a 8 variable 2 compartment model of a pyramidal cell from the CA3 region of the hippocampus of the brain

- Strongly Nonlinear
- Highly Coupled
- Multi-dimensional

We will use this neuronal model to demonstrate control of the responses of a complex system

Target different spiking behaviors

i.e. states with different Inter Spike Intervals (ISI)

So $\mathcal{P} \equiv ISI, \quad \mathcal{P}^\star \equiv ISI^\star$

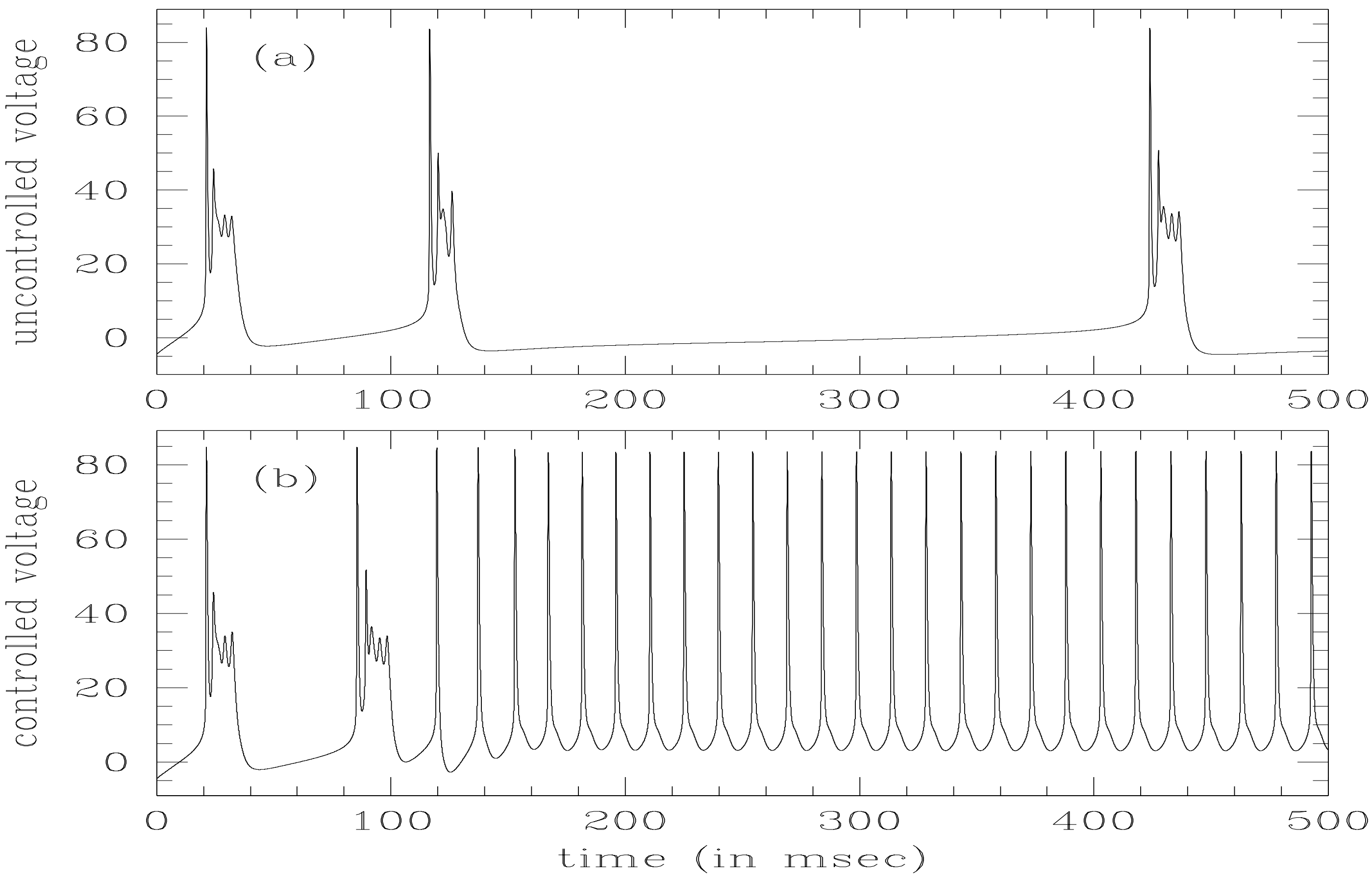
The parameter most accessible to quick external manipulation is the applied soma current i_s

Procedure for reaching and maintaining a particular ISI, by adjusting the applied current i_s via adaptive feedback is as follows:

$$i_s(n+1) = i_s(n) - \gamma(ISI_n - ISI^\star)$$

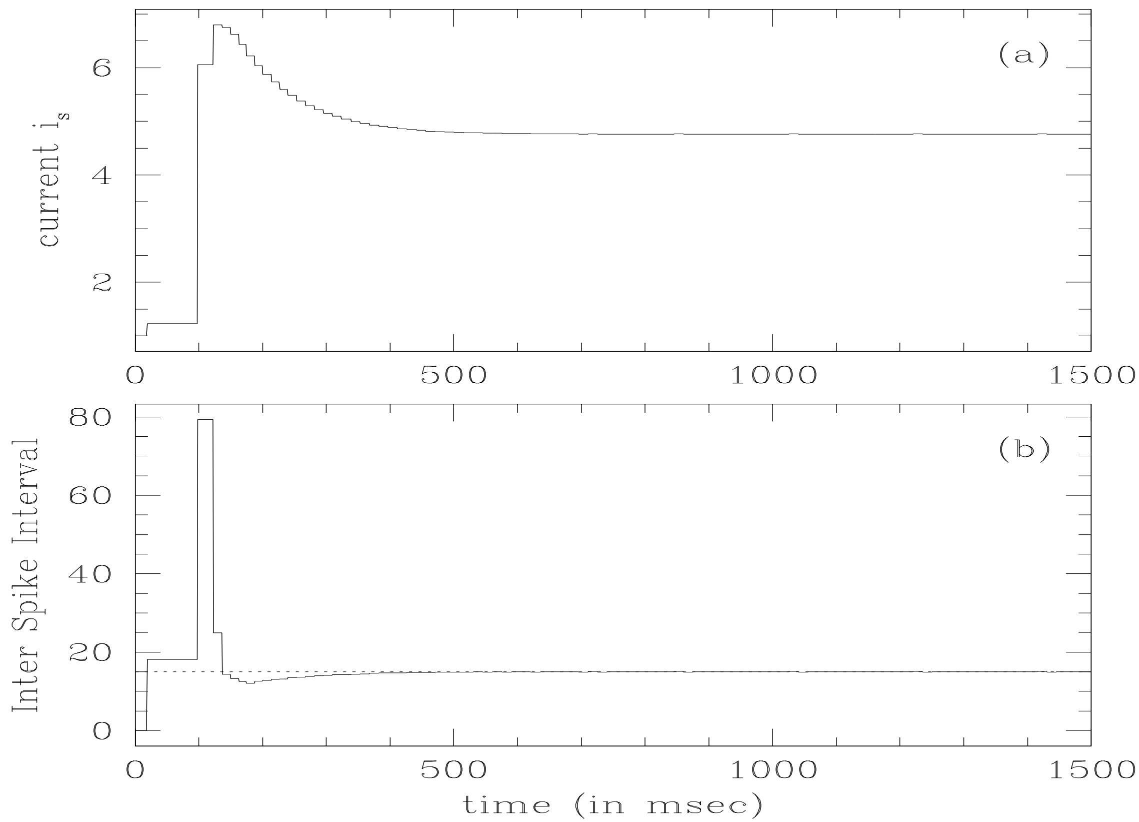
where ISI_n is the current inter spike interval

i.e. the time difference between the current spike and its immediately preceding one



(a) Uncontrolled Neuron

(b) Under Feedback Control : with target $ISI^* = 15$ msec



(a) Time evolution of the soma current i_s
(b) Time evolution of the Inter Spike Interval
Dashed line : target ISI of 15 msec

- This control algorithm has the desired effect of tuning the value of i_s such that :

  the dynamics of the combined equations yields a steady state with $ISI = ISI^{\star}$

- The control algorithm does not require a priori knowledge of the governing equations of the system

- The only information necessary to implement adaptive control is the current ISI value

  i.e. the difference in the time at which the current spike occurs and that at which the previous one had occured

- Once the system achieves the target :

  It remains there and the control equation is switched off
  As the error signal is zero

- If the parameters begin to drift (for instance, due to environmental fluctuations) :

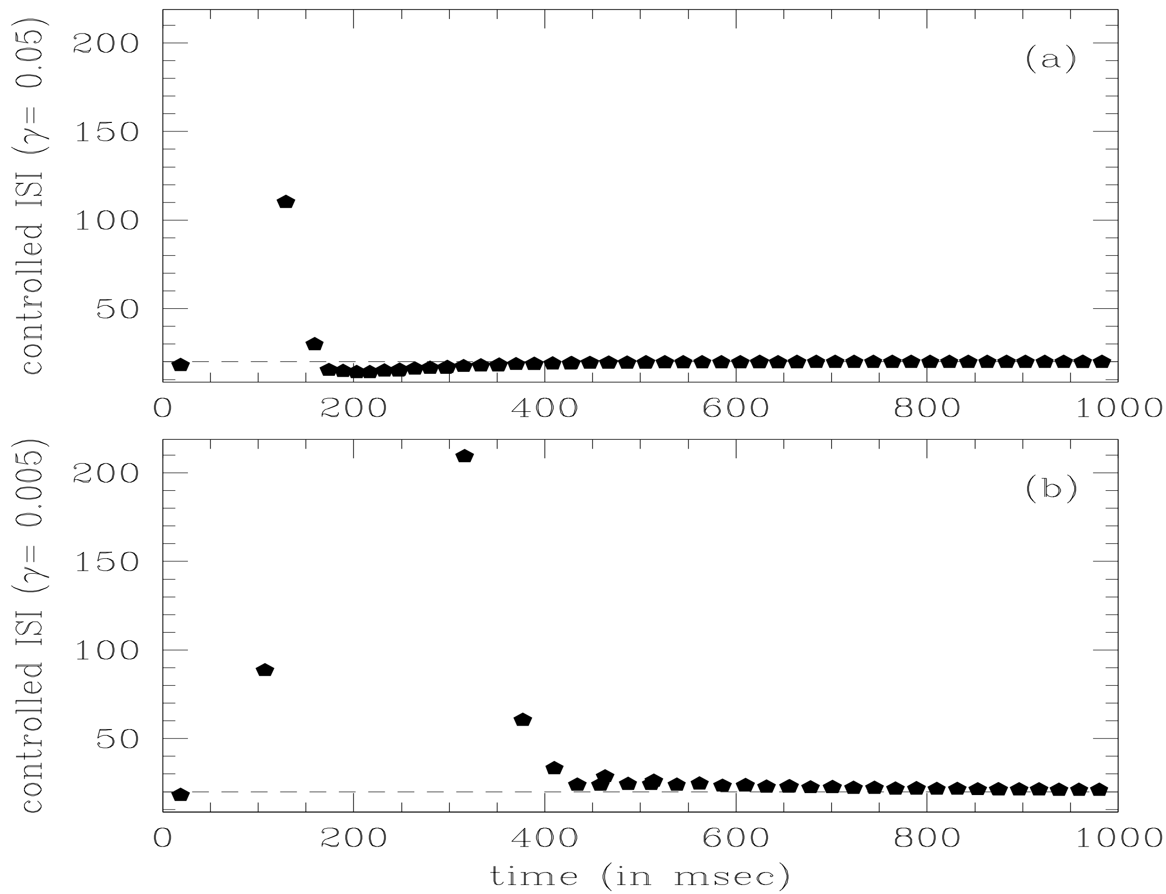
  The control automatically becomes effective again
  As the error signal becomes non zero again

  And this readily brings the system back to the desired state

The stiffness γ determines how rapidly the system is controlled

The control (recovery) time : defined as the time required to reach the desired state

Crucially depends on the value of γ



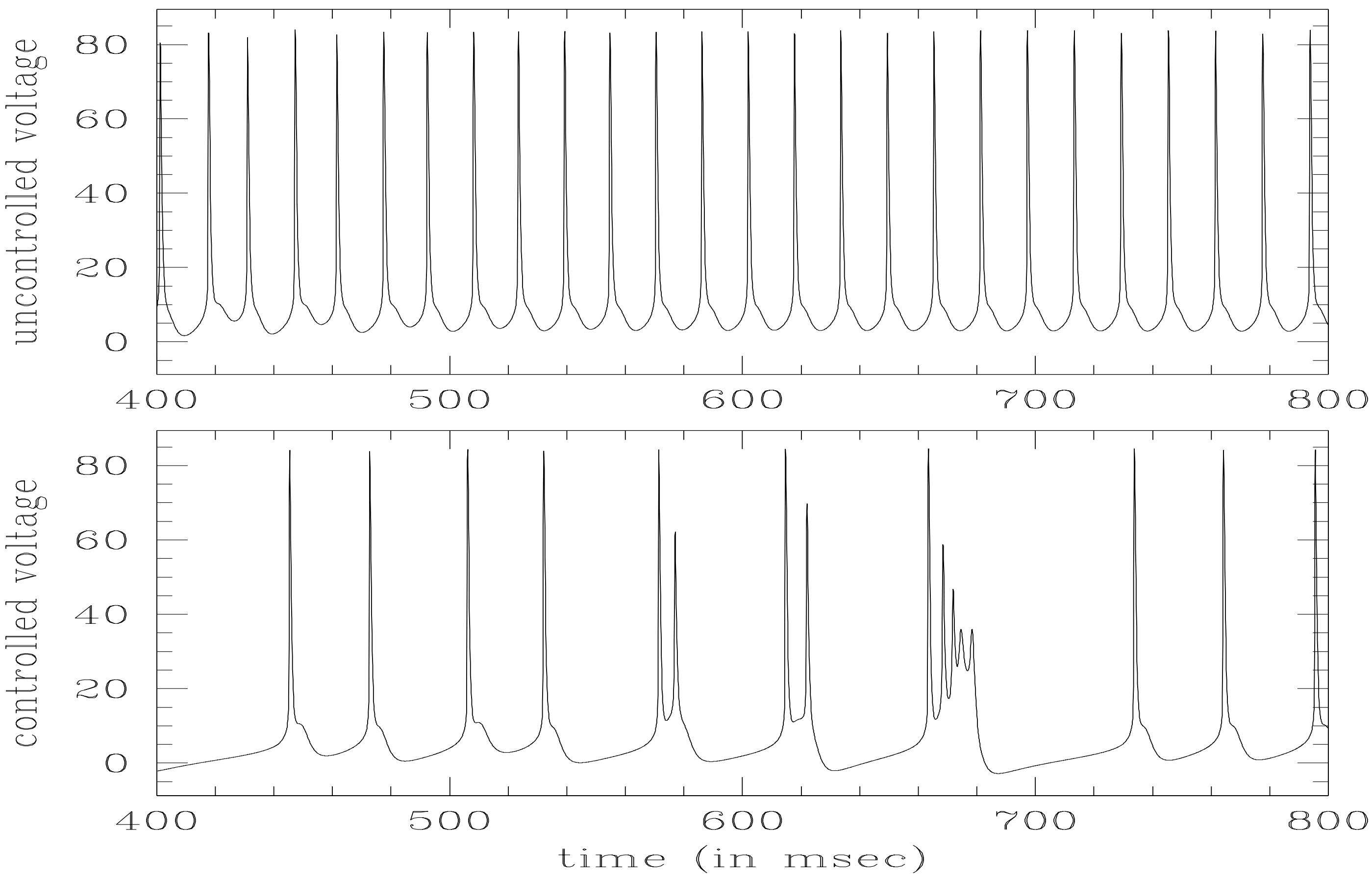
Time evolution of the controlled Inter Spike Interval :
With stiffness of control γ : (a) 0.05 and (b) 0.005

Targetting an irregular firing state :

Set a large target ISI (> 30 msec)

As the system can only support irregular firing beyond that ISI, the adaptive mechanism leads to fluctuating current i_s, which in turn leads to irregular firing around a mean ISI^*

Thus we can achieve the desired effect of obtaining a state with very irregular spikes



(a) Uncontrolled neuron

(b) Under feedback anti-control

Robustness ?

In real experiments it is conceivable that the ISI may not measured very accurately

In order to be useful the technique should be robust with respect to noise in ISI determination

Checked that the method indeed is successful even if the ISI information in the feedback loop has a noisy spread amounting upto 5 percent of the targetted ISI

- Caveat: If the system does not have any parameter regime yielding the targetted dynamical behaviour – adaptive control will fail
  So the method is capable of achieving only those targets that have a stable basin of attraction somewhere in parameter space
- Not too limiting, as nonlinear systems generically support many different dynamical behaviours in different parameter regimes :
  evident from the rich bifurcation structure in parameter space of nonlinear systems
- Adaptive control then works like an efficient search algorithm for varied dynamical characteristics in parameter space

Controlling Extended Systems

A wealth of complex patterns have been observed in a variety of extended systems :

- Chemically Reacting Systems
- Nonlinear Optics
- Oscillating Fluid Surfaces and Granular Layers
- Electroconvection in Liquid Crytals
- Coupled josephson junction arrays
- Morphogenesis, Self replication of living cells
- Cardiac tissue and Neural systems
- Population dynamics

Thus control techniques capable of stabilising complex patterns are of much potential use

We will now show that adaptive control techniques are sufficiently general and versatile, and are capable of achieving spatio-temporal targets of wide-ranging complexity :

- Spatio-temporal fixed points
- Spatial patterns
- Spatio-temporal Chaos

Demonstrate this control principle on a
2-dimensional lattice of coupled logistic maps

This system is capable of exhibiting a rich variety of spatio-temporal patterns as well as spatio-temporal chaos:

Thus it provides a good testing ground for the technique

Note that the method is quite general and can be directly applied to other extended systems as well

Evolution equations:

$$x_{n+1}(i,j)) = f(\alpha, x_n(i,j)) + \frac{\epsilon}{4}\sum_{\text{nn}}\{g(x_n(i_{nn}, j_{nn}) - g(x_n(i,j))\}$$

where

- nn denotes the 4 nearest neighbours of site (i,j)
- Local map $f(x) = 1 - \alpha x^2$ with α indicating the strength of the nonlinearity
- Parameter ϵ gives the strength of coupling among neighbours
- Different coupling forms used: e.g. $g(x) = x$ and $g(x) = f(x)$

Controlling Spatio-temporal fixed points

Target: Synchronised lattice – with each element invariant in time as well

Situations where such a control is relevant, include the maintenance of steady states in biophysical processes under fluctuating environmental conditions :

- Biological Thermostats
- Regulation of Cell Reactions
- Maintenance of Homeostasis (i.e. the relative constancy of the internal environment with respect to blood pressure, pH, blood sugar, osmolarity and electrolytes)

To reach and maintain a stable spatio-temporal fixed point :

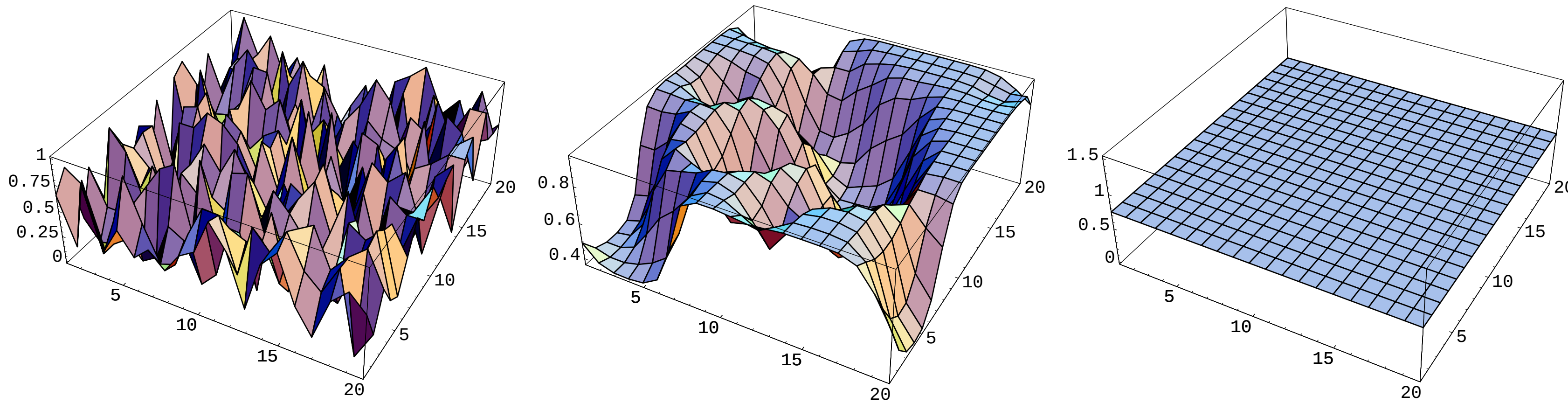
Desired value of all lattice sites $x(i)$ is $x^\star$ at all times

Then the control equation has $\mathcal{P} \equiv x$ and $\mathcal{P}^\star \equiv x^\star$

$$\alpha_{n+1} = \alpha_n - \gamma(x_n(i_c, j_c) - x^\star)$$

where (i_c, j_c) is the single site chosen for monitoring feedback

Note that the controlled parameter α is changed globally here



The random initial lattice with parameter value far from what yields the target :

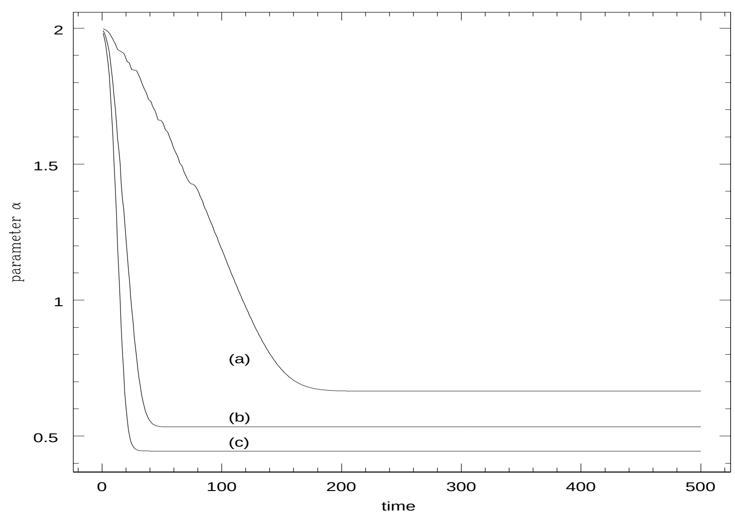
Under control dynamics rapidly reaches the desired spatio-temporal state

The stiffness γ determines how rapidly the system is controlled

Numerical experiments show :

- For small γ , recovery time is inversely proportional to the stiffness of control
- Recovery time not dependent on lattice size
- Recovery time not dependent on dimensionality of the lattice

Variation of the controlled parameter as a function of time



Using temporal feedback for control to a spatio-temporal fixed point

Stiffness of Control γ is : (a) 0.01   (b) 0.05   (c) 0.1

In certain applications one may want to stabilise such spatial patterns

To target spatial patterns we must use spatial feedback

This is obtained by measuring the local neighbourhood of a monitored site

The feedback has to be specifically tailored according to the distinguishing characteristics of the desired targetted pattern

Demonstrate this for the case of two distinct patterns : the chequerboard (squares) and stripes

In order to target chequerboard patterns, one can use its simplest characteristic, which is the requirement that

$$x(i,j) - x(i+1, j-1) = 0$$
$$x(i,j) - x(i-1, j+1) = 0$$
$$x(i,j) - x(i+1, j+1) = 0$$
$$x(i,j) - x(i-1, j-1) = 0$$

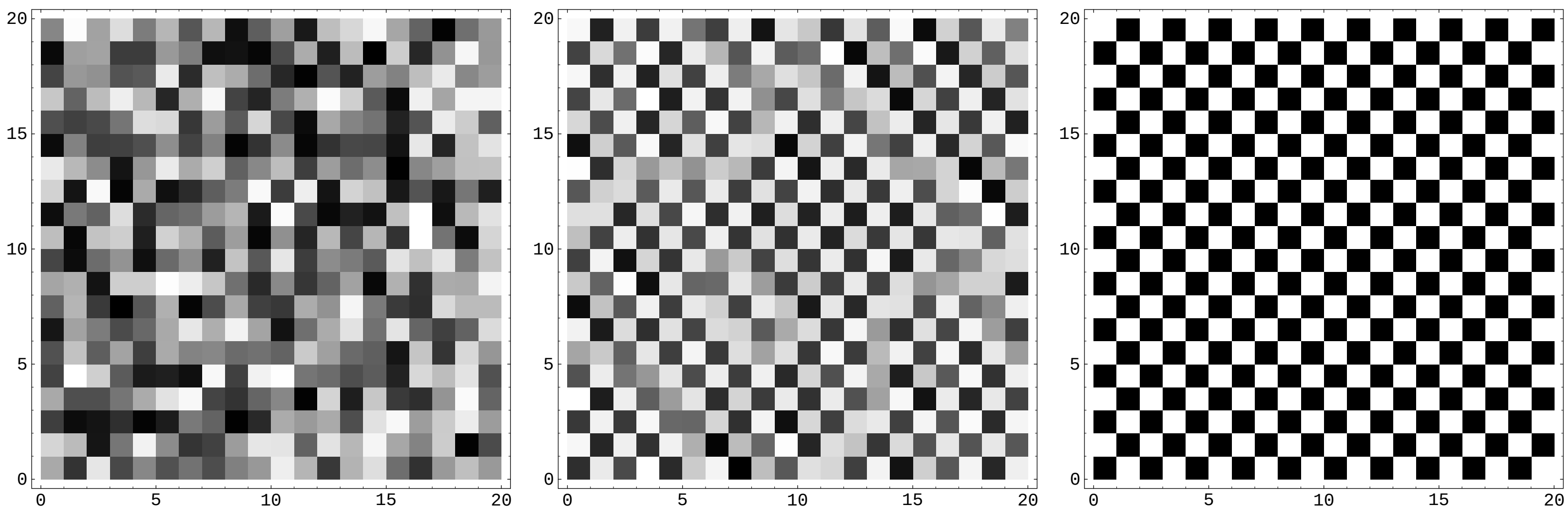
for all i,j

Utilizing the above to construct an error signal:

$$\Delta x = |\{x(i_c, j_c) - x(i_c+1, j_c-1)\} + \{x(i_c, j_c) - x(i_c-1, j_c+1)\}$$
$$+\{x(i_c, j_c) - x(i_c+1, j_c+1)\} + \{x(i_c, j_c) - x(i_c-1, j_c-1)\}|$$

where (i_c, j_c) is the site monitored for feedback

Controlling to a spatial chequerboard pattern by using spatial feedback



Control Equation :

$$\alpha_{n+1} = \alpha_n - \gamma \Delta x$$

If one wanted to target a striped pattern the demand is:

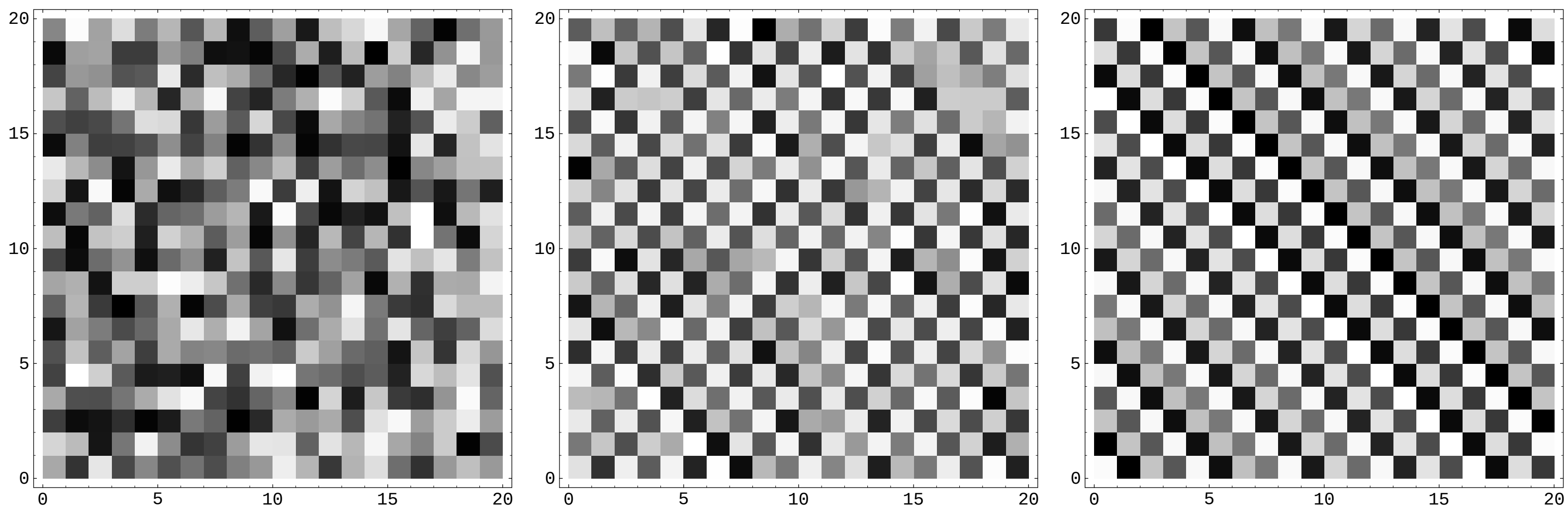
$$x(i,j) - x(i+1, j-1) = 0$$
$$x(i,j) - x(i-1, j+1) = 0$$

This gives the following error signal :

$$\Delta x = |\{x(i_c, j_c) - x(i_c + 1, j_c - 1)\} + \{x(i_c, j_c) - x(i_c - 1, j_c + 1)\}|$$

where (i_c, j_c) is the site monitored for feedback

Controlling to a spatial striped pattern by using spatial feedback



Spatial periodicity achieved by targetting spatial patterns does not necessarily imply temporal periodicity :

As feedback does not have any temporal information here and no specific temporal pattern is demanded by the control mechanism

- Control method drives the lattice to the targetted patterns very effectively
- One obtains the first (stable) configuration which satisfies the demand of error being zero
- Driven by spatial gradients, the parameter evolves in a manner such that the desired spatial correlations emerge
- In a sense then, varied pattern formation occurs in this augmented dynamical system, dictated by the driving equation for the parameter(s)

Targetting Spatio-temporal Chaos

Another application of practical importance is in enhancing spatio-temporal chaos

Examples: Mixing Flows and Chemical Reactions – where the Enhancement of Chaos leads to Improved Performance

Possible biological applications as well : Neural Systems

If the desired state is chaotic rather than periodic, one needs to choose an appropriate property $\mathcal{P}$ which reflects the chaotic nature of the target state

An appropriate adaptive strategy is to take $\mathcal{P}$ to be the instantaneous local stretching rate Δx, in space or time

The local stretching Δx in time is given by

$$\Delta x = |x_n(i_c, j_c) - x_{n-1}(i_c, j_c)|$$

where (i_c, j_c) is the site monitored for feedback

One can also use a spatial feedback

For instance, one can demand local spatial roughness (or local stretch in space) :

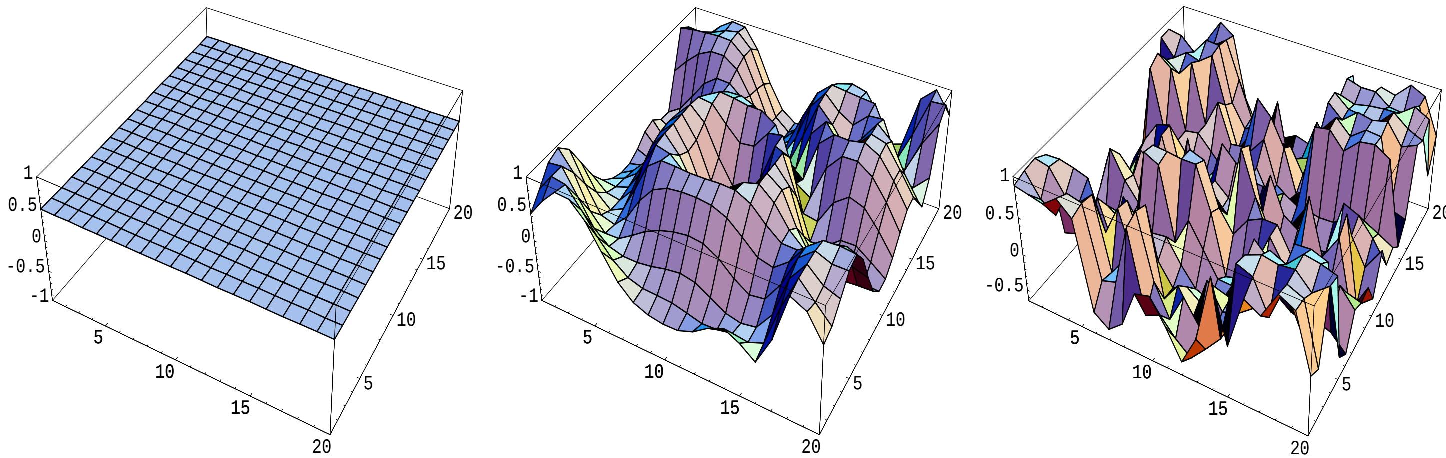
$$\Delta x = |\sum_{\mathrm{nn}} x_n(i_c, j_c) - x_n(i_{nn}, j_{nn})|$$

where nn denotes the 4 nearest neighbours of the monitored site (i_c, j_c)

When target $\Delta x = 0$: spatio-temporal fixed point is obtained

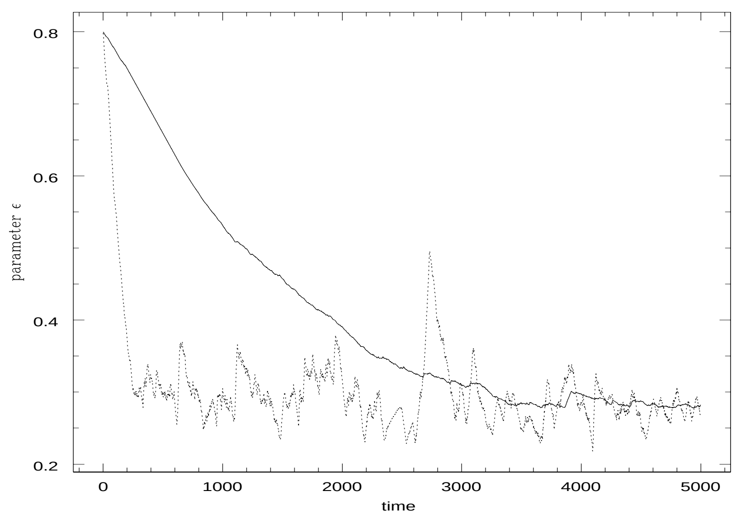
When target Δx is large : leads the system to spatiotemporal chaos

Control Equation : $\alpha_{n+1} = \alpha_n + \gamma(\Delta x_{target} - \Delta x)$



The controlled parameter rapidly evolves in time to a suitable range and then fluctuates within a range of values, so as to keep the targetted stretch rate, on an average, satisfied

Variation of the controlled parameter as a function of time



Target : spatio-temporal chaos
The stiffness of control γ is 0.001 (——) and 0.01 (. . . .)

The range of values within which the parameter fluctuates increases with increasing stiffness γ

Outlook

We have presented several adaptive algorithms : utilizing both spatial and temporal feedbacks

The techniques are rapid, powerful and robust

We have applied the scheme to successfully achieve a wide range of spatio-temporal targets, from synchronisation and spatial patterns to spatio-temporal chaos

These techniques then have the potential for application in systems such as coupled oscillator systems, chemical reactions and Josephson junction arrays

Significant features of these methods are:

- They work with limited information of the state of the system
- None of the control algorithms required a priori knowledge of the governing equations of the system

Since they can be implemented without explicit knowledge of the dynamics, which can be treated effectively as a black–box :

Useful in experimental applications

- The only information necessary to implement adaptive control (or adaptive anti-control) is either the difference between the current value of a variable and its previous value or the value of the monitored sites and a suitable set of neighbours
- One arbitrarily chosen site in the bulk of the lattice (and/or its local neighbourhood) is monitored for measuring the error signal

  Thus only one site provides the global feedback which drives the entire lattice to the target
- So the schemes are not measurement intensive

Adaptive Feedback Control is a versatile tool for controlling inherently chaotic systems to a variety of target states