What is Geometry?

(an introduction to modern geometry)

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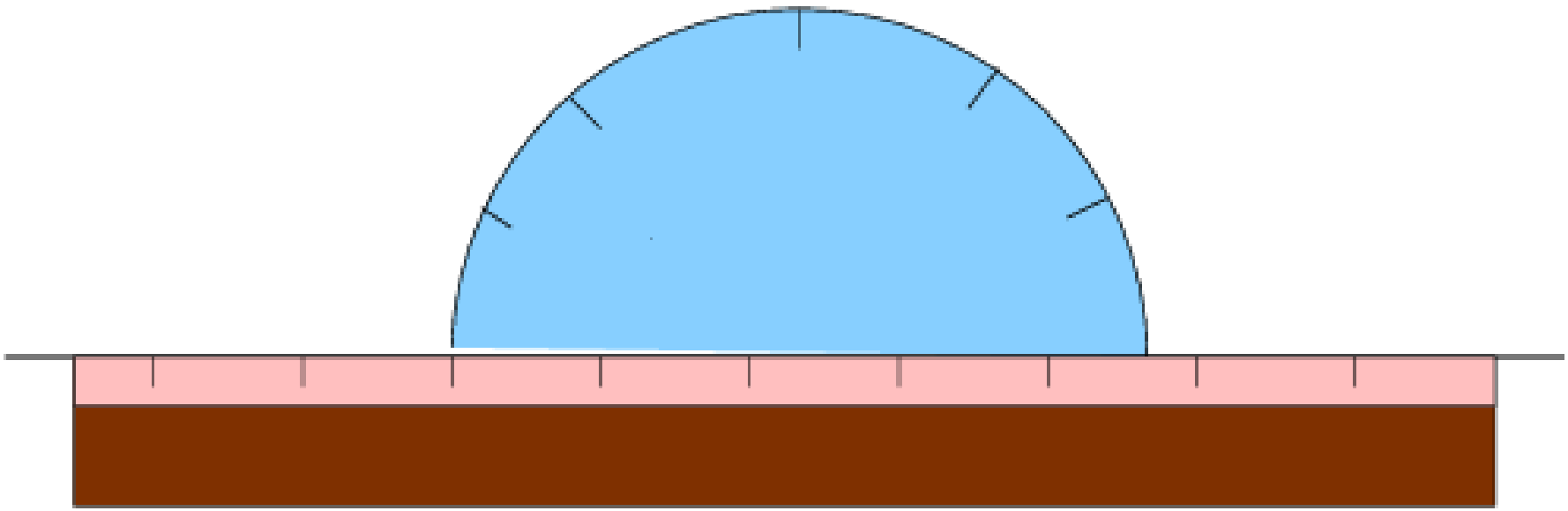
"The rest" is how mathematicians usually refer to non-mathematicians!

Ruler and Protractor Placement



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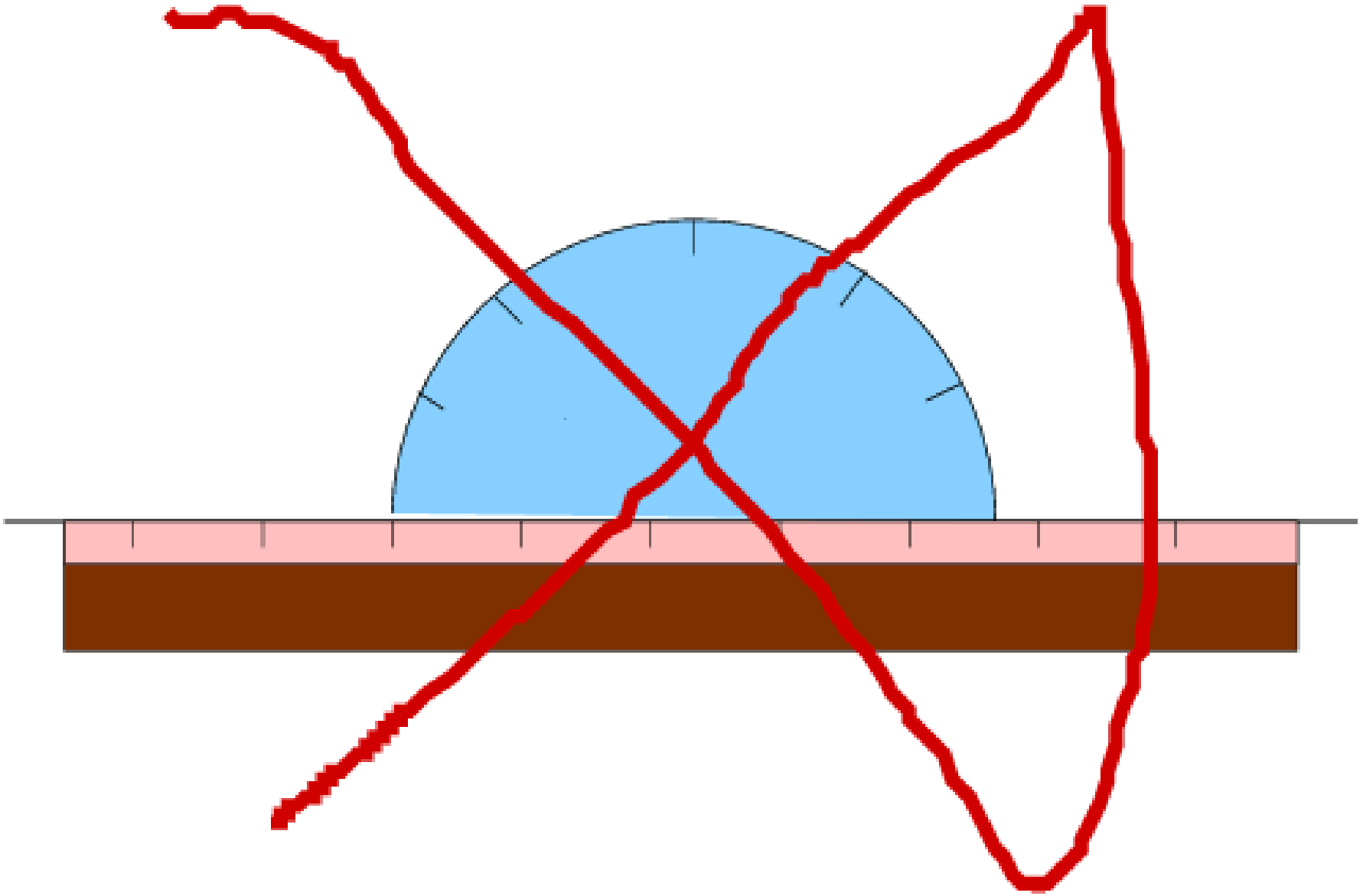

Numbers and Geometry

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Pictures and Geometry

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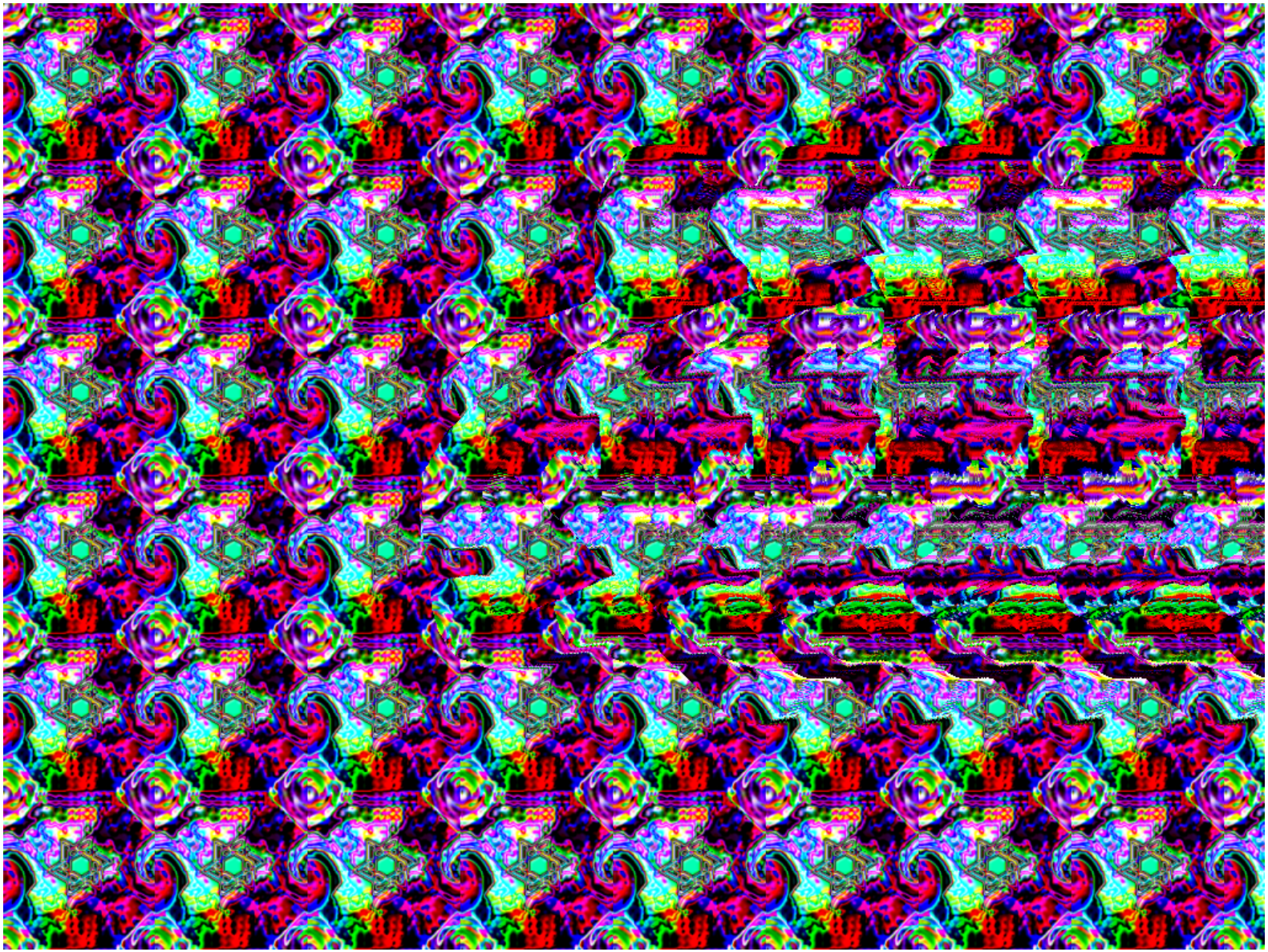
Do you see it?



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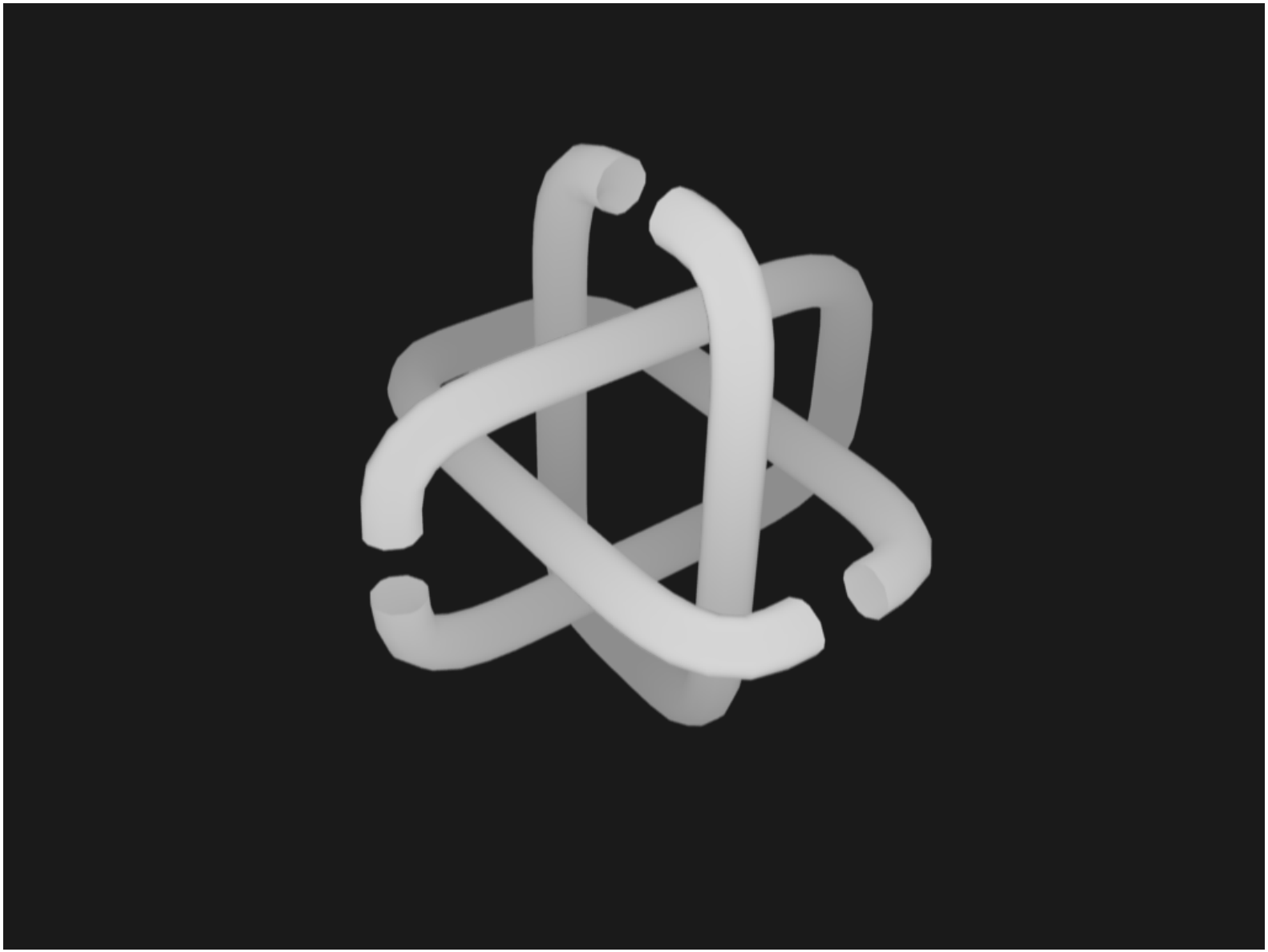

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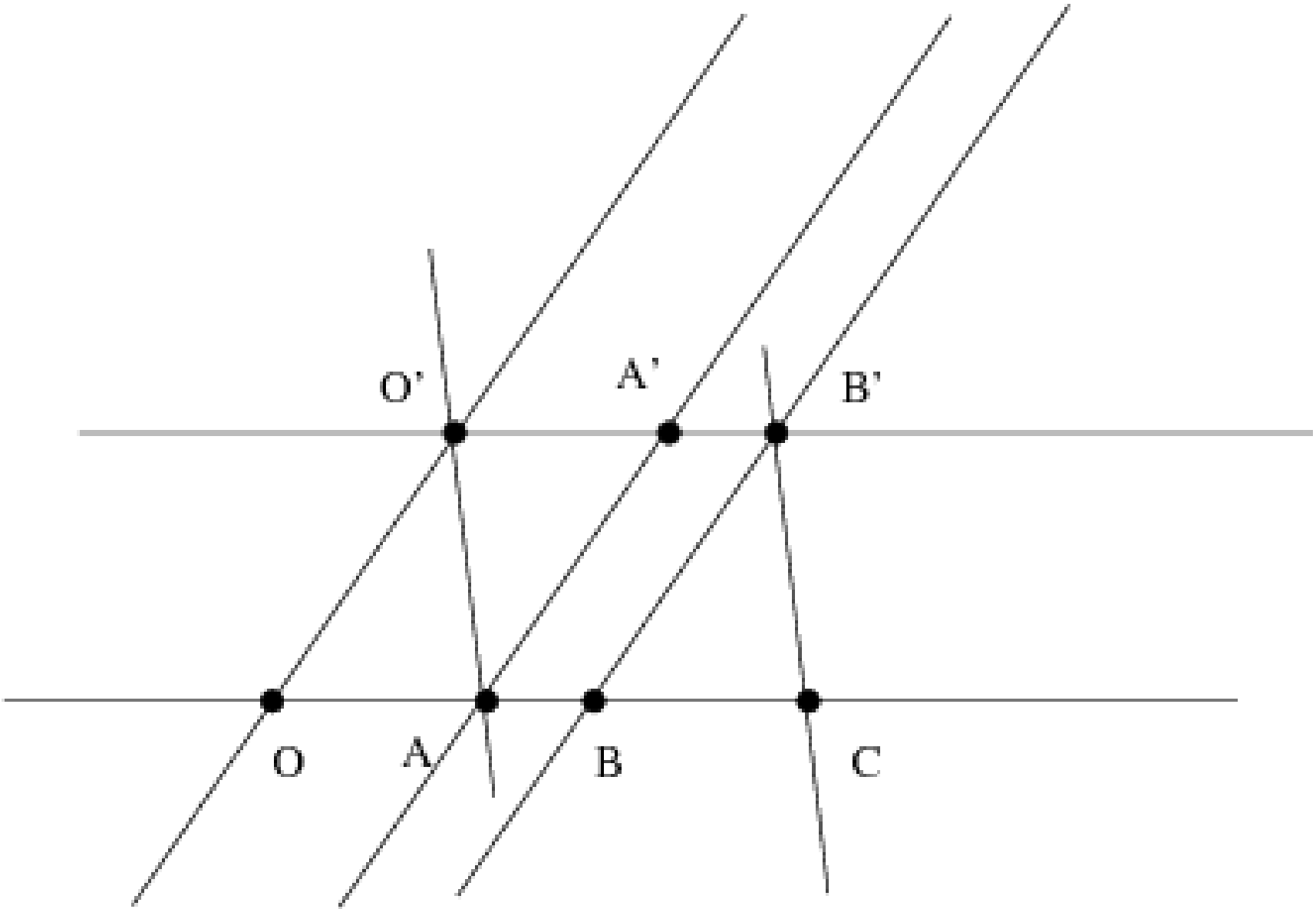
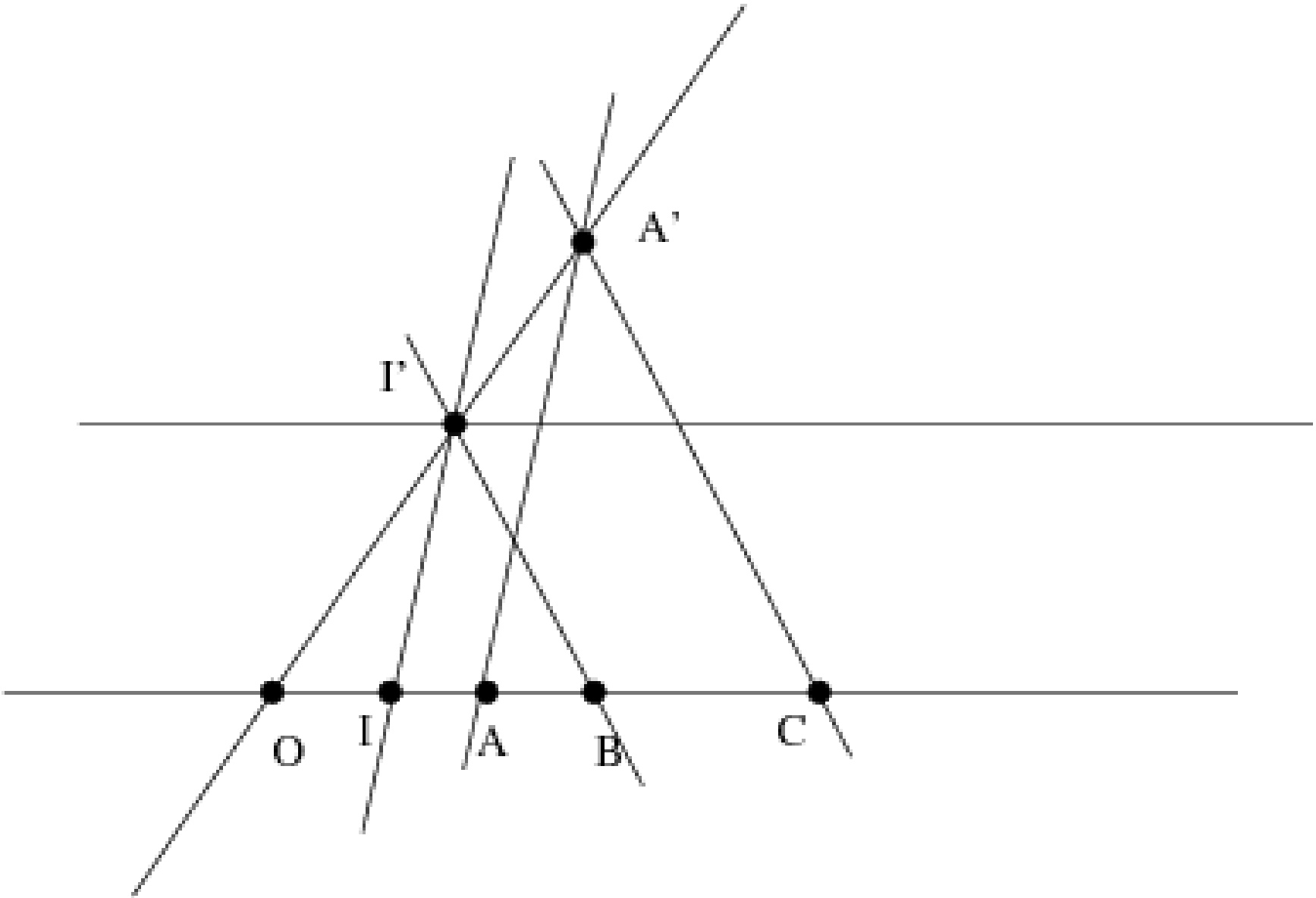
Divisibility: Given any segment we can chop it up into two (or more) equal parts.

Arithmetic



Arithmetic



Euclid's Fifth Postulate

Given a line and a point outside it, there is a unique line through that point which does not meet the given line.

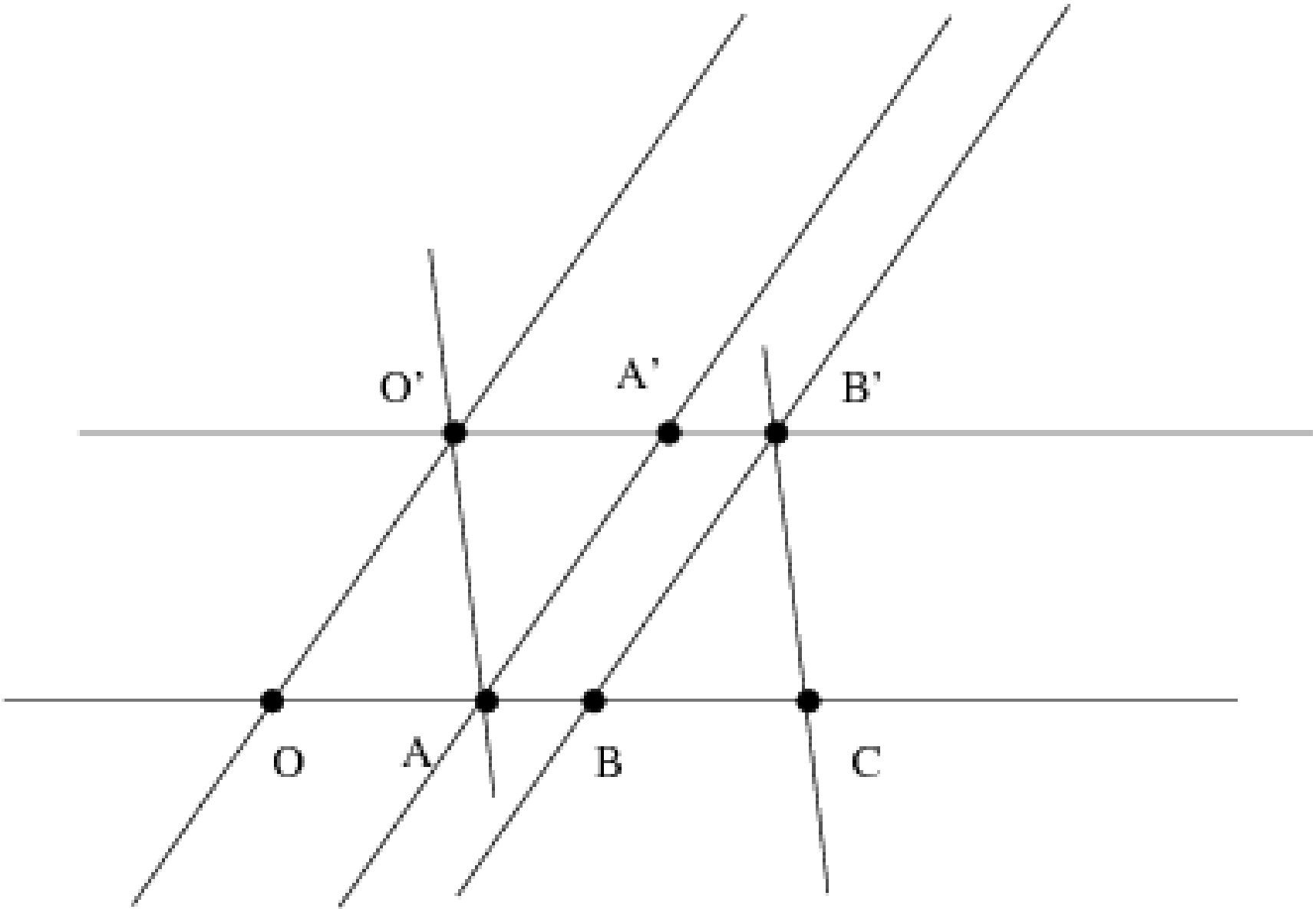
Euclid's Fifth Postulate

Given a line and a point outside it, there is a unique line through that point which is parallel to the given line.

Euclid's Fifth Postulate

All coplanar lines meet!

Projectivise



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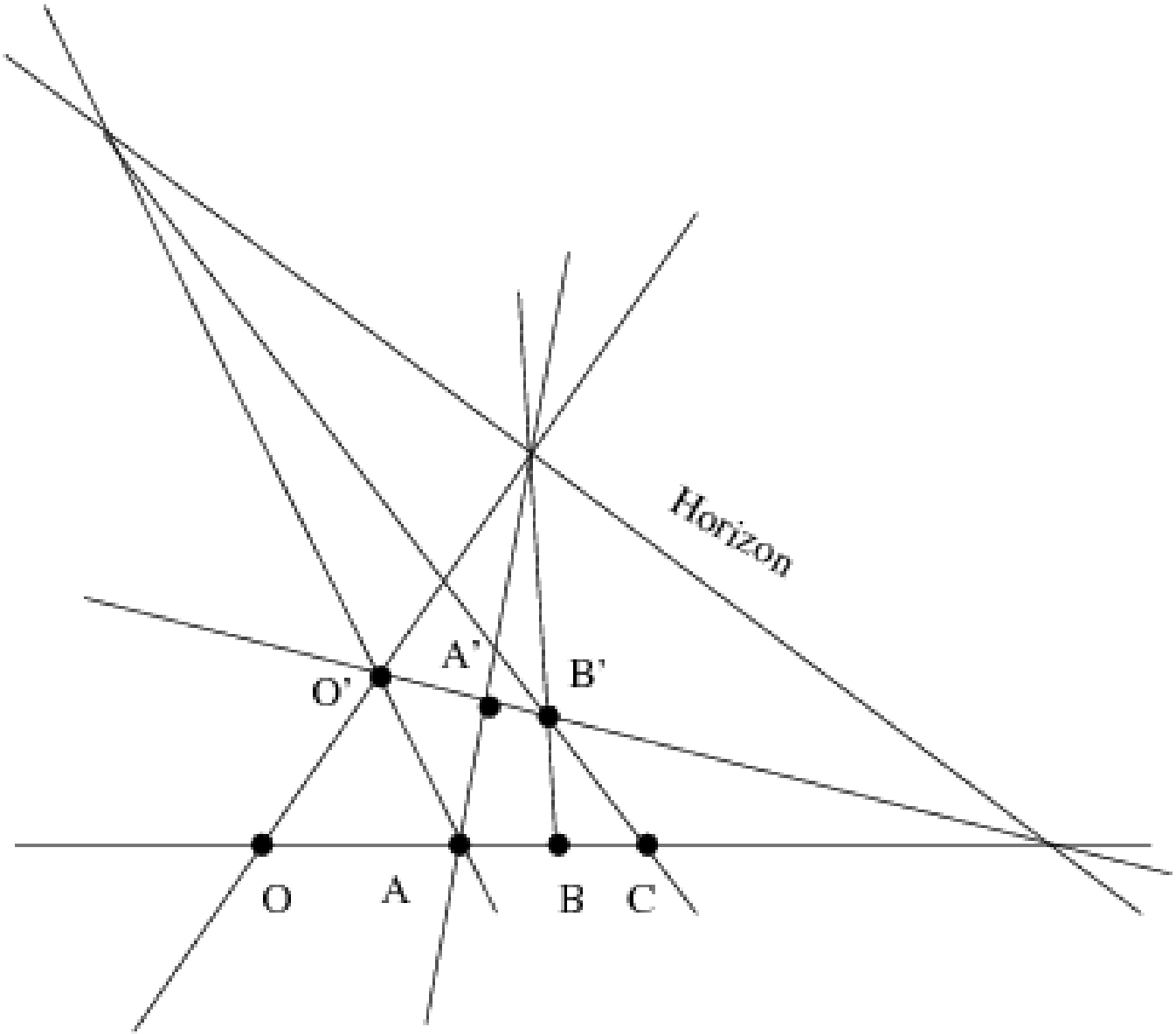

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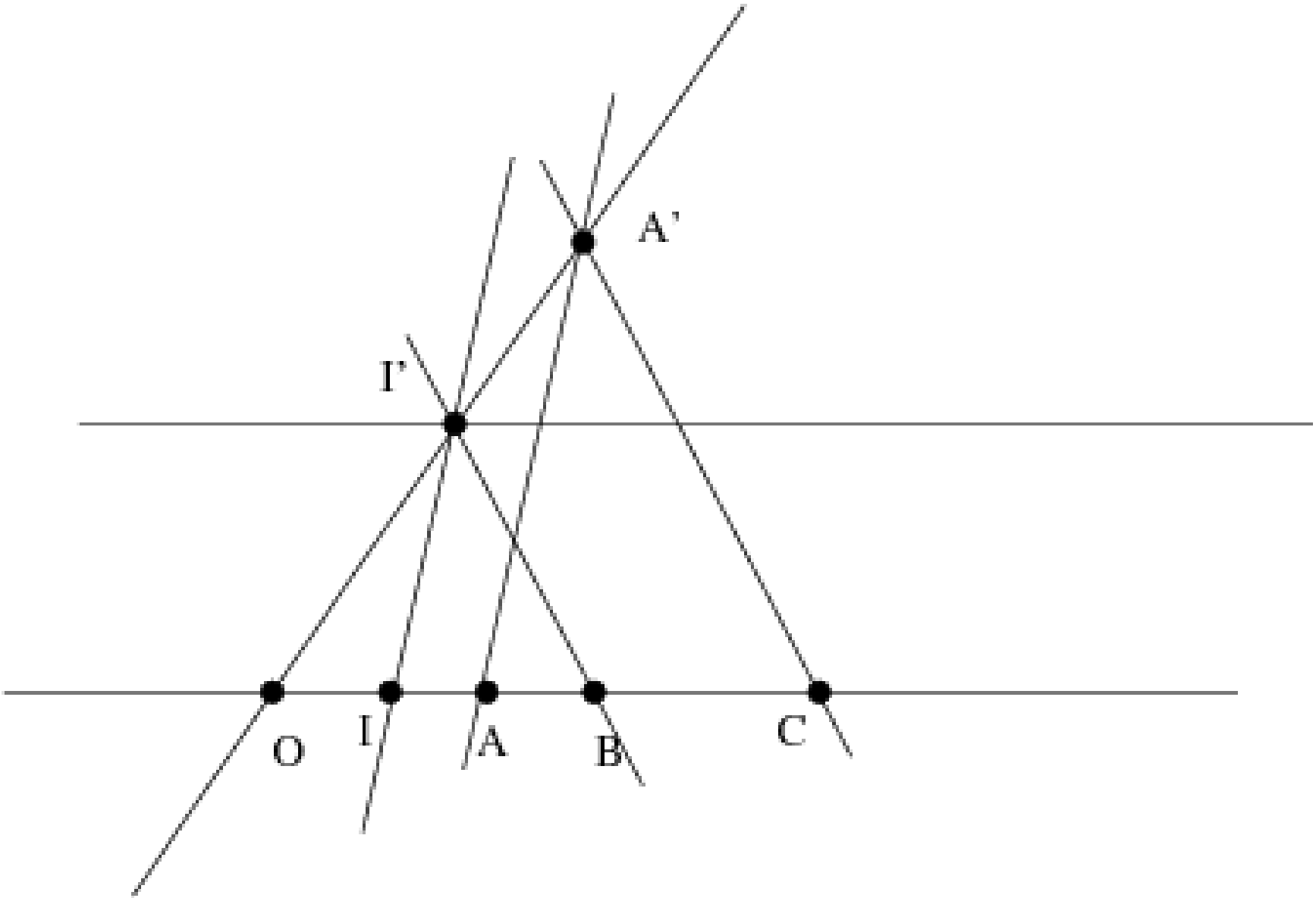

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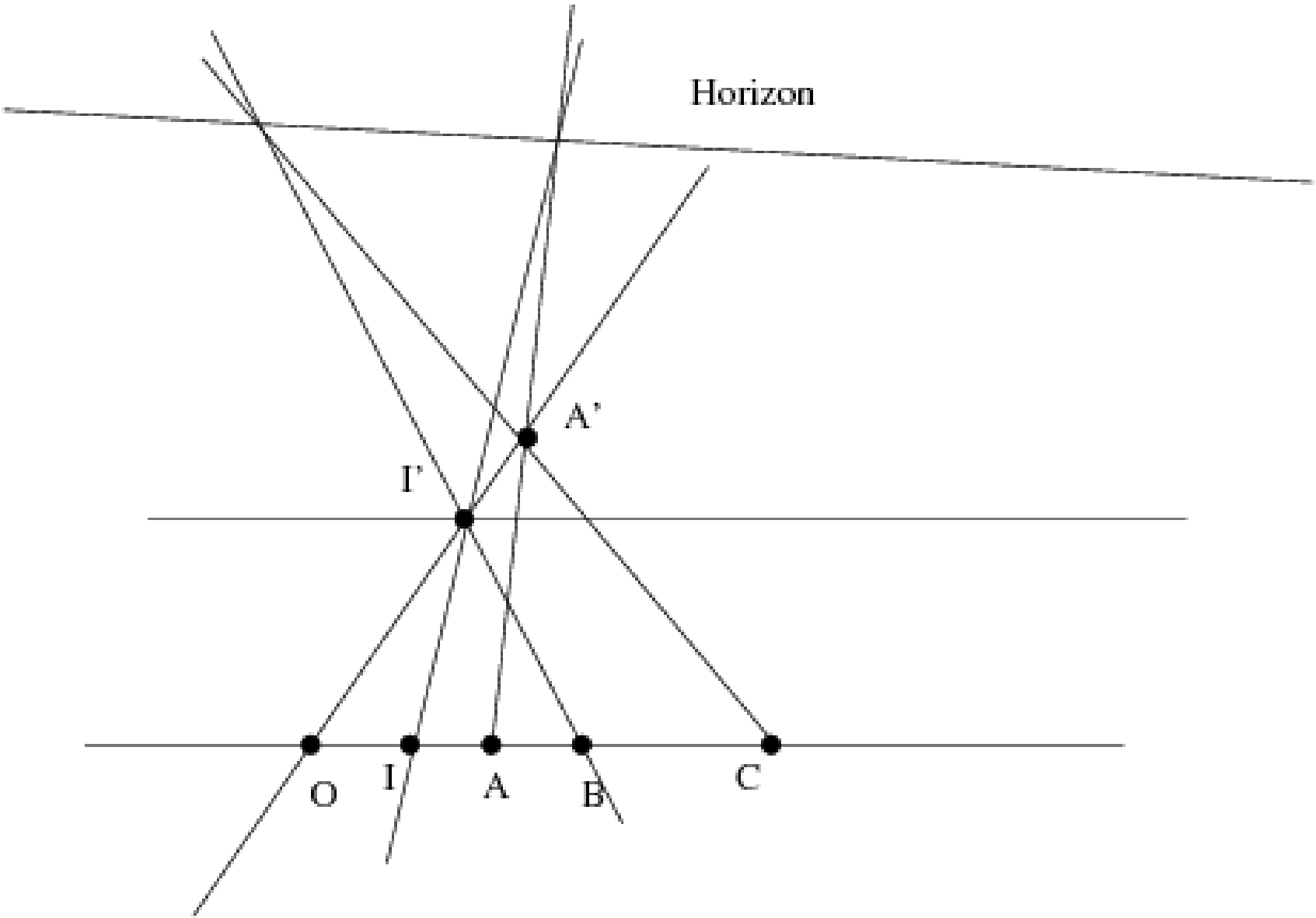
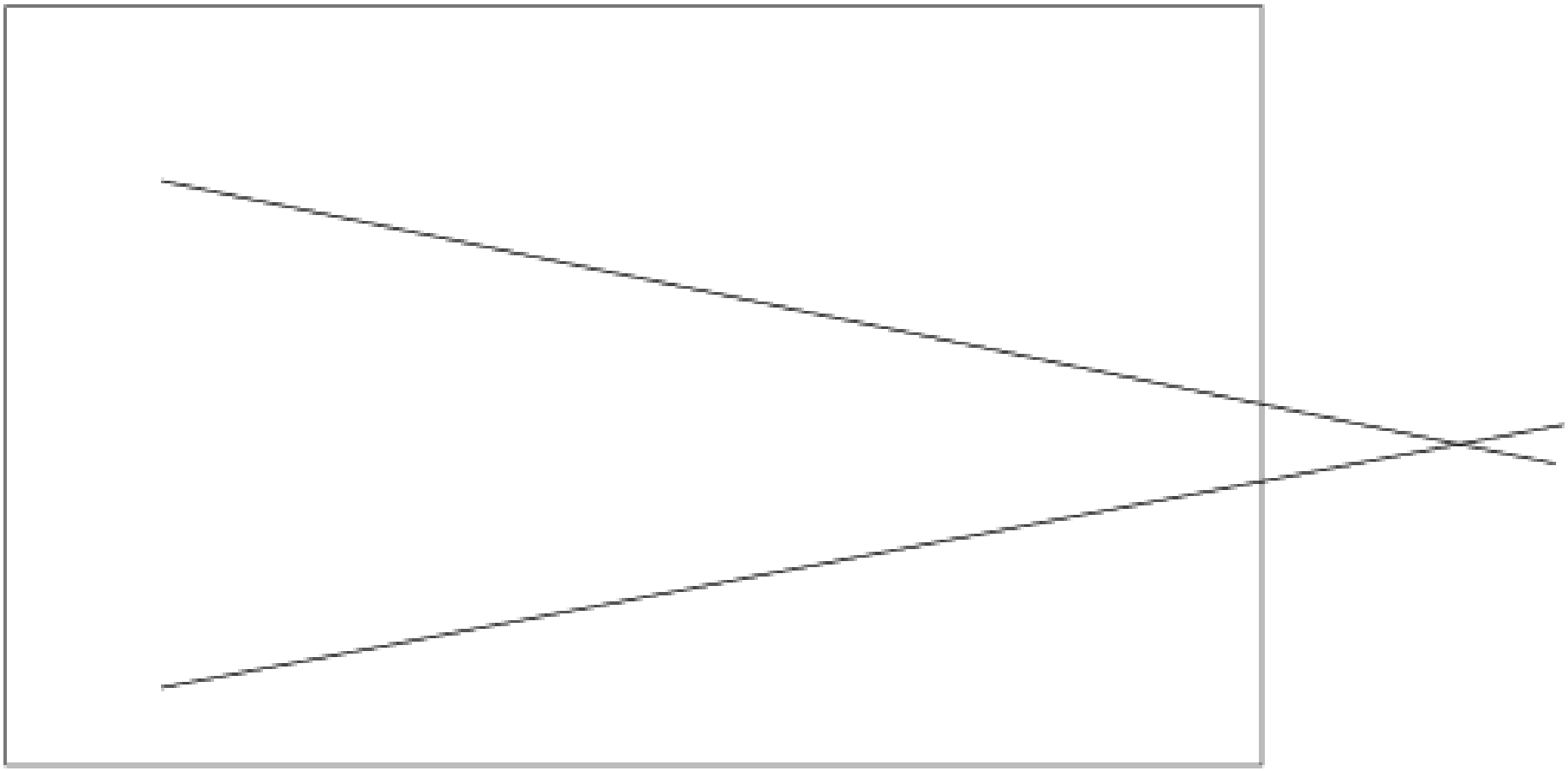


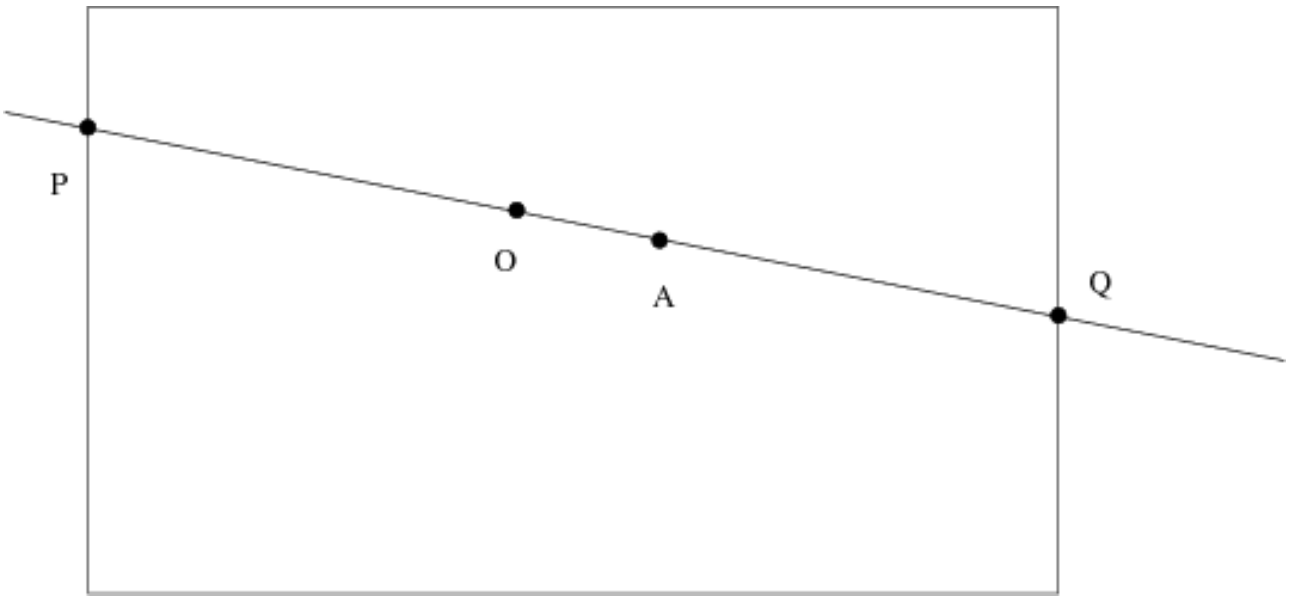
Geometry on a Piece of paper



Non-Euclidean Fifth Postulate

Given a line and a point outside it, there are infinitely many lines through that point which do not meet the given line.

Non-Euclidean Distance



$$\log(|AP|/|AQ|) - \log(|OP|/|OQ|)$$

Thank you for your attention